REMAINING FATIGUE LIFE ANALYSIS:
A COMPARISON BETWEEN CONVENTIONAL
ASPHALT CONCRETE - DENSE GRADED (CAC-DG) AND
ASPHALT-RUBBER HOT MIX - GAP GRADED (ARHM-GG)

BY

LUTFI RAAD
Associate Professor of Civil Engineering
Transportation Research Center
Institute of Northern Engineering
University of Alaska Fairbanks
Fairbanks, AK 99775

STEPHAN SABOUNDJIAN
Graduate Student
Transportation Research Center
Institute of Northern Engineering
University of Alaska Fairbanks
Fairbanks, AK 99775

and

JOHN CORCORAN
President
Manhole Adjusting Inc.
2300 Greenwood Ave.
P.O. Box 250
Monterey Park, CA 91754

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ABSTRACT

A procedure for estimating the remaining fatigue life of existing bituminous pavements is developed. This procedure incorporates laboratory fatigue data for tensile strain or surface curvature in terms of repetitions to failure. It also utilizes laboratory data on reduction of flexural stiffness with load repetitions. The proposed method is applied to investigate the remaining fatigue life of a conventional asphalt concrete mix - dense graded (CAC-DG) and an asphalt-rubber hot mix - gap graded (ARHM-GG) using laboratory flexure fatigue data and multilayer elastic analysis of typical pavement sections. Results show that for a given initial state of fatigue damage, the remaining fatigue life of ARHM-GG could be significantly longer than CAC-DG. It is also illustrated that thinner sections of ARHM-GG, compared with CAC-DG, will exhibit the same remaining fatigue life. This reduction in thickness becomes more significant with increasing foundation support under the pavement surface layer.
INTRODUCTION

The use of asphalt-rubber binder in pavements has progressed from its application in asphalt-rubber and aggregate membranes (ARAM), also referred to as stress absorbing membranes (SAM) and asphalt-rubber and aggregate membrane (ARAM) interlayers, also referred to as stress absorbing membrane interlayers (SAMI) that started about 1968 to the incorporation of asphalt-rubber hot mix in pavement overlays that began in 1975 (1,2,3). In addition to overlays, asphalt-rubber mixtures have been used as a surface course in reconstructed pavement sections (4). Field performance data on asphalt-rubber hot mix pavements indicate significant improvement in fatigue resistance, abrasion resistance, and resistance to aging in comparison with conventional asphalt concrete mixtures (5,6). These field data support laboratory fatigue test results that illustrate improved fatigue and fracture properties of asphalt-rubber mixtures in comparison with conventional asphalt concrete (7,8,9). Results of a recent study by Raad et al. (8) show that overlay thickness determinations using laboratory fatigue data support recommendation guidelines proposed by the California Department of Transportation (Caltrans) on overlay thickness equivalencies between asphalt-rubber hot mix - gap graded (ARHM-GG) and conventional asphalt concrete - dense graded (CAC-DG). Although asphalt-rubber overlays seem to provide a cost effective option for pavement rehabilitation (2,7), the determination of the remaining fatigue life of the existing pavement needs to be assessed for the purpose of improved overlay thickness
selection. This would be essential particularly in lieu of research presented by Seebaly et al. (10) indicating a lag between structural capacity reduction and surface cracking of field test sections. In this case, the backcalculated moduli of the asphalt layer were reduced by 50% before lineal cracking, or AASHO Class 2 and 3 cracking were observed on the pavement surface. Structural damage in terms of reduction of the modulus of the bituminous surface does therefore occur prior to any visual fatigue cracking in the pavement. Such deterioration could be assessed through non-destructive testing using, for example, the Falling Weight Deflectometer (FWD).

In this paper, a procedure for estimating the remaining fatigue life of existing bituminous pavements is summarized. The proposed method utilizes flexure fatigue data for ARHM-GG and CAC-DG. The remaining life is expressed in terms of the reduction of the modulus of the pavement surface and the applied wheel load and does not require knowledge of previous wheel load magnitudes and repetitions. In this respect, the proposed method provides a definite advantage over current procedures that use Miner’s cumulative damage hypothesis (11) to estimate remaining life. Fatigue criteria are expressed in terms of 1) flexure tensile strains, and 2) surface curvature. Results are used to compare the fatigue behavior of ARHM-GG and CAC-DG pavements. Specifically, the number of load repetitions required to induce a given degree of fatigue damage are compared. In addition, the remaining fatigue life of ARHM-GG and CAC-DG pavements are determined for similar sections with the same degree of fatigue damage (i.e. equal reduction in surface layer modulus). Thickness
equivalencies between ARHM-GG and CAC-DG pavements associated with fatigue performance are also established using both strain and curvature criteria.

EXPERIMENTAL WORK

Materials

CAC-DG and ARHM-GG beam specimens were obtained from new pavement sections that were constructed in California for the purpose of comparing the field performance of CAC-DG and ARHM-GG materials. The crumb rubber material used is scrap tire, vulcanized, with specific gravity of 1.15-1.20, containing a minimum of 25 percent natural rubber. All materials meet Caltrans specifications. A summary of specifications for aggregate gradations, asphalt-rubber binder properties, and CAC-DG and ARHM-GG properties is presented in Tables 1 and 2.

Fatigue Testing

The fatigue behavior of CAC-DG and ARHM-GG was investigated using controlled strain flexure beam testing. Detailed description of the testing procedure and equipment is presented elsewhere (8). The beam specimens were cut to about 2 in. by 2 in. by 15 in. and were loaded at 5 in. interval third points. The density of the CAC-
DG specimens varied between 152 lb/cuft and 153 lb/cuft whereas the density of ARHM-GG specimens varied in the range of 146 lb/cuft and 148 lb/cuft. All tests were conducted using MTS closed loop hydraulic testing equipment and a haversine displacement pulse having a width of 0.10 sec and a frequency equal to 60 cpm. Fatigue tests were performed in an environmental chamber and the temperature of the specimens was maintained between 70°F and 73°F. For a given displacement pulse, the variation of applied load, and tensile and compressive strains across center of beam specimen, were monitored with number of pulse applications. Fatigue failure was assumed to occur when the flexure stiffness E determined from central beam deflections and applied load using simple beam theory assumptions reduced by 50%.

Results

Fatigue criteria were developed in terms of 1) maximum tensile strain at middle beam section, and 2) central beam curvature, with number of load repetitions to failure. These criteria are shown in Figure 1. The strain criterion is expressed as:

For CAC-DG,

\[ N_f = 1.471 \times 10^{-10} \left( \frac{1}{\varepsilon_r} \right)^{4.55} ; \quad (r^2 = 0.93) \]  (1)
For ARHM-GG,

\[ N_f = 2.350 \times 10^{-12} \left( \frac{1}{\varepsilon_t} \right)^{5.41} ; \quad (r^2 = 0.93) \]  

(2)

where,

\[ \varepsilon_t = \text{tensile strain} \]
\[ N_f = \text{number of repetitions to failure} \]

The curvature criterion, on the other hand, can be written in terms of curvature (\( \rho \)) or the inverse of the radius of curvature (1/R) at the center of the beam as:

For CAC-DG,

\[ N_f = 8.232 \times 10^{-12} \left( \frac{1}{\rho} \right)^{4.94} ; \quad (r^2 = 0.90) \]  

(3)

For ARHM-GG,

\[ N_f = 4.205 \times 10^{-13} \left( \frac{1}{\rho} \right)^{5.68} ; \quad (r^2 = 0.92) \]  

(4)
where, $\rho = (\varepsilon_t + \varepsilon_c)/h$; $h =$ beam thickness in inches; $\varepsilon_t, \varepsilon_c$ are tensile and compressive strains at top and bottom of middle beam section respectively.

It is interesting to note that the curvature criterion incorporates the thickness of the beam in the fatigue equation. This implicitly implies that for a given strain condition, thicker beam sections will exhibit longer fatigue life. This is consistent with fatigue fracture propagation described by Paris law and applied to asphalt concrete and asphalt-rubber hot mix (7).

Fatigue damage associated with a given number of load applications can be described as a loss in flexural stiffness. The reduction in flexural stiffness, tensile stiffness, and compressive stiffness for both CAC-DG and ARHM-GG as deduced from the fatigue tests is illustrated in Figure 2. Tensile and compressive stiffness in this case are determined from the applied load and the measured tensile and compressive strains at the top and bottom of the middle section of the beam specimen. As shown, the loss in stiffness seems to proceed at a much faster rate after $10^3$ repetitions for CAC-DG in comparison with ARHM-GG where the decrease proceeds at a slower rate. Best fit correlations of the stiffness reduction defined as the ratio of flexural stiffness $E$ after $N$ load applications to the initial flexural stiffness $E_i$ are developed in terms of tensile strain $\varepsilon$, and beam curvature $\rho$ as follows:
For CAC-DG,

\[
\frac{E}{E_t} (\varepsilon)^{1/4} = 0.19056 - 0.01964 \log N; \quad (r^2 = 0.74)
\] (5)

\[
\frac{E}{E_t} (\rho)^{1/4} = 0.18832 - 0.01919 \log N; \quad (r^2 = 0.73)
\] (6)

For ARHM-GG,

\[
\frac{E}{E_t} (\varepsilon)^{1/4} = 0.17226 - 0.01509 \log N; \quad (r^2 = 0.76)
\] (7)

\[
\frac{E}{E_t} (\rho)^{1/4} = 0.17288 - 0.01501 \log N; \quad (r^2 = 0.77)
\] (8)

The above results of stiffness reduction $E/E_t$ are shown in Figures 3 and 4. These results indicate that for a given repetitions of $\varepsilon$ or $\rho$, the reduction in $E$ will be slightly higher for CAC-DG in comparison with ARHM-GG thereby exhibiting more fatigue damage.
REMAINING LIFE ANALYSIS

The remaining fatigue life for a pavement with a given $E/E_t$ is defined as the number of repetitions of a given load (expressed in terms of tensile strain or surface curvature) required to induce ultimate fatigue damage by reducing $E/E_t$ to 0.50. The remaining fatigue life can be estimated using the laboratory determined relations for $E/E_t$ in terms of load repetitions and applied tensile strain or surface curvature (Equations 5-8). For a given fatigue damage, expressed as $E/E_t$, these equations can be used to determine the equivalent number of repetitions $N_e$ of a given load provided the corresponding load induced strain or curvature is known. In this case, a remaining fatigue life factor $R_f$ could be defined as:

$$R_f = 1 - \frac{N_e}{N_f} \quad \text{(9)}$$

where $N_e$ and $N_f$ are both determined from Equations 5-8 depending on the mix type (i.e. CAC-DG or ARHM-GG) and strain or curvature criterion applied. $N_f$ is the number of repetitions to failure for a new pavement estimated by substituting $E/E_t = 0.50$. 

Rand et al.
For CAC-DG,

\[ R_f = 1 - 10^{-\left( \frac{E_{E_1} - 0.50}{0.01964} \right) \gamma_{\text{us}}} \]  \hspace{1cm} (10)

\[ R_f = 1 - 10^{-\left( \frac{E_{E_1} - 0.50}{0.01919} \right) \rho_{\text{us}}} \]  \hspace{1cm} (11)

For ARHM-GG,

\[ R_f = 1 - 10^{-\left( \frac{E_{E_1} - 0.50}{0.01509} \right) \gamma_{\text{us}}} \]  \hspace{1cm} (12)

\[ R_f = 1 - 10^{-\left( \frac{E_{E_1} - 0.50}{0.01501} \right) \rho_{\text{us}}} \]  \hspace{1cm} (13)

The corresponding remaining fatigue life \( N_f \) can then be expressed as:
For strain criterion,

\[ N_f = R_f \cdot (A1) \cdot \left( \frac{1}{e} \right)^{A2} \] (14)

For curvature criterion,

\[ N_f = R_f \cdot (B1) \cdot \left( \frac{1}{\rho} \right)^{B2} \] (15)

Where A1, A2, B1, and B2 are material constants defined in Equations 1-4.

The variation of the remaining life factor \( R_f \) with \( E/E_i \), \( e_i \), and \( \rho \) is shown in Figures 5 and 6. Results indicate that \( R_f \) increases with increase in \( E/E_i \), \( e_i \), and \( \rho \) and attains slightly larger values for ARHM-GG in comparison with CAC-DG. The limits of variation are between zero and 1 by definition.

Remaining fatigue life prediction requires the assessment of \( E/E_i \) at any given period during the service life of the pavement. \( E/E_i \) could be determined through backcalculation procedures using non-destructive pavement deflection equipment such as the FWD. Response parameters in terms of strains or surface curvature associated with a given wheel load are then used, together with \( E/E_i \), to determine the remaining fatigue life as described in Equations 10-15. It should be noted that surface curvature
can be easily estimated from surface deflection data. This could provide direct estimation of pavement remaining life following FWD measurement of a simulated wheel load. The estimation of load repetitions required to induce a given fatigue damage in CAC-DG or ARHM-GG in terms of stiffness reduction $E/E_i$ could also be obtained by substituting $(1-R_r)$ for $R_r$ in Equations 14 and 15.

Remaining fatigue life analysis has been investigated using multilayer elastic theory and the proposed material models. Typical three layer pavements with CAC-DG and ARHM-GG surfaces were analyzed using ELSYM5 (12) computer program. A summary of the cases considered is presented in Table 3. A standard 9000 lb wheel load with tire pressure equal to 100 psi was used. The analysis aimed at comparing the fatigue performance of CAC-DG and ARHM-GG pavements. Specifically, the number of load repetitions required to induce a given reduction in surface layer modulus, the number of load repetitions associated with remaining fatigue life for a given state of fatigue damage $E/E_i$, and the equivalent thickness correlation between CAC-DG and ARHM-GG were investigated.

Results of analysis are presented in Figures 7-15. The number of wheel load repetitions $N_d$ required to cause a given fatigue damage represented by stiffness reduction $E/E_i$ is generally larger for ARHM-GG than CAC-DG. This difference increases with increase in surface layer thickness, underlying base and subgrade
support, and degree of fatigue damage (i.e. decreasing $E/E_i$) (Figures 7-10). A similar trend is observed for the variation of remaining fatigue life for a pavement with a given $E/E_i$. In this case, the difference in remaining fatigue life between CAC-DG and ARHM-GG pavements increases with increasing thickness and underlying pavement support but decreases with increase in initial degree of fatigue damage (Figures 11-14). Results of analysis were also used to establish layer thickness equivalencies between CAC-DG and ARHM-GG layers. In this case, the two materials are assumed to have initially the same fatigue damage state (i.e. $E/E_i$). Layer thicknesses for both CAC-DG and ARHM-GG are then selected to provide equal remaining fatigue life. As illustrated in Figure 15, thinner sections of ARHM-GG will exhibit the same remaining fatigue life in comparison with CAC-DG pavements. The reduction in thickness is more significant for pavements with higher base and subgrade moduli. For example, assuming the base and subgrade moduli equal to 80 ksi and 20 ksi respectively, and a CAC-DG surface layer with thickness in the range of 6 in. and 10 in., then the corresponding equivalent ARHM-GG thickness will be between 2 in. and 5 in. respectively. In case of overlay applications, more support will be provided by the underlying pavement in comparison with that provided by the base and subgrade in a typical new pavement structure. It is therefore expected that the equivalent ARHM-GG overlay thickness could be significantly smaller as demonstrated by Raad et al. (8).
SUMMARY AND CONCLUSIONS

In this paper a method is proposed for predicting the remaining fatigue life of existing bituminous pavements. This method incorporates laboratory fatigue data for tensile strain or radius of curvature in terms of repetitions to failure. It also utilizes laboratory data on reduction of flexural stiffness with load repetitions. The proposed method is applied in the analysis of conventional asphalt concrete - dense graded (CAC-DG) and an asphalt-rubber hot mix - gap graded (ARHM-GG) pavements. Controlled-strain fatigue test data for CAC-DG and ARHM-GG were incorporated in the analysis. Results indicate that the number of load repetitions required to induce a given fatigue damage, expressed in terms of the ratio of layer modulus after a given period of service to its initial modulus prior to the incurrence of any fatigue damage, could be significantly larger for ARHM-GG than CAC-DG. Similar observations are made for the remaining fatigue life of CAC-DG and ARHM-GG assuming a given initial state of fatigue damage. Thickness equivalencies between CAC-DG and ARHM-GG based on equal remaining fatigue life of pavement sections with similar initial fatigue damage are developed. It is illustrated that thinner sections of ARHM-GG will be required for a given remaining fatigue life in comparison with CAC-DG pavements. This reduction in thickness becomes more significant with increasing foundation support under the pavement surface layer which warrants consideration of using ARHM-GG in pavements and particularly in overlays.
The results presented in this paper are based on laboratory tests and simple multilayer elastic analysis covering limited loading and temperature conditions. Field research is needed to calibrate the proposed models and to verify the conclusions and trends obtained.

ACKNOWLEDGMENTS

This work has been supported by a research grant from Manhole Adjusting Inc. This help is gratefully acknowledged. The authors would like to thank Dr. Nick Coetzee who reviewed the research work and provided feedback and guidance.
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<table>
<thead>
<tr>
<th>Sieve Size</th>
<th>Percent Passing</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Per Specification</td>
<td>Actual Field Results</td>
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<tr>
<td></td>
<td>CAC Dense Graded</td>
<td>ARHM Gap Graded</td>
<td>CAC Dense Graded</td>
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<tr>
<td>3/4 in.</td>
<td>100</td>
<td>100</td>
<td>100</td>
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<tr>
<td>1/2 in.</td>
<td>95-100</td>
<td>90-100</td>
<td>97</td>
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<tr>
<td>3/8 in.</td>
<td>80-95</td>
<td>78-92</td>
<td>91</td>
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<td>No. 4</td>
<td>59-66</td>
<td>28-42</td>
<td>68</td>
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<td>43-49</td>
<td>15-25</td>
<td>53</td>
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<td>No. 30</td>
<td>22-27</td>
<td>5-15</td>
<td>35</td>
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<tr>
<td>No. 200</td>
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<td>2-7</td>
<td>13</td>
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<tr>
<td>Asphalt Binder %</td>
<td>5.2 - 6.5</td>
<td>7.5 - 8.7</td>
<td>6.2</td>
</tr>
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Note:
Asphalt Cement (AR-4000) in CAC-DG

Components of Asphalt-Rubber Binder:
AR-4000 Asphalt Cement
2% - 6% Asphalt Modifier (by total weight of Asphalt -Rubber binder)
78% - 82% Asphalt Cement and Modifier
18% - 22% Rubber

** ARHM-GG Proposed Standard Specifications for Public Works Construction, Section 203-11.3
<table>
<thead>
<tr>
<th>Property</th>
<th>Specification Limits</th>
<th>Actual Average Results of Binder Tested</th>
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<tr>
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<td>1350 - 3050</td>
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<tr>
<td>Centipoise (ASTM D2669)</td>
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<td>Penetration, Cone at 77 °F in 1/10 mm</td>
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<td>51</td>
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<tr>
<td>(ASTM D217)</td>
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<tr>
<td>Resilience 77 °F in Percent Rebound</td>
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<td>(ASTM D3407)</td>
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<tr>
<td>Field Softening Point in °F</td>
<td>125 - 165</td>
<td>142</td>
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<td>(ASTM D36)</td>
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Table 3: Summary of Material Properties and Pavement Cases Studied

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<thead>
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<th>PROPERTY</th>
<th>MATERIAL</th>
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<tr>
<td></td>
<td>CAC-DG</td>
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<tr>
<td>Modulus, ksi</td>
<td>$E_1 = 550$</td>
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<td></td>
<td>$E/E_1 = 1, .85, .75, .65, .55$</td>
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<tr>
<td>Poisson's Ratio</td>
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<td>Thickness, in.</td>
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Note: $E_1 =$ Initial Modulus of Surface Layer  
$E = $ Modulus of Surface Layer after N Repetitions  
$E_b =$ Modulus of Base Layer  
$E_s =$ Modulus of Subgrade
STRAIN CRITERION

- - ARHM-GG - - CAC-DG

REpetitions to failure

CURVATURE CRITERION

- - ARHM-GG - - CAC-DG

REpetitions to failure

Fig. 1: Fatigue Failure Criteria for CAC-DG and ARHM-GG
**CAC-DG**

Applied Deflection = 0.0183 in

**ARHM-GG**

Applied Deflection = 0.0180 in

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**Fig. 2**: Variation of Stiffness with Load Repetitions for CAC-DG and ARHM-GG.
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CAC-DG; $E_b/E_s = 80/20$

**STRAIN CRITERION**

![Graph showing variation of $N_d$ with CAC-DG thickness for different $E/E_i$ ratios.]

**CURVATURE CRITERION**

![Graph showing variation of $N_d$ with CAC-DG thickness for different $E/E_i$ ratios.]

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**STRAIN CRITERION**

**ARHM-GG THICKNESS, IN**

**CURVATURE CRITERION**

**Fig. 14**: Variation of $N_f$ with ARHM-GG Surface Layer
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